# Different ways to describe a plane in $\mathbb{R}^3$

$$2x - 4y + 6z = 8$$

Cartesian Equation

$$\begin{aligned} x &= 4 + 2s - 3t \\ y &= s \\ z &= t \end{aligned}$$

Parametric equation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = 8$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

Vector Equation

through $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$, and with unit normal vector $\frac{1}{\sqrt{56}} \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$

through $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Three Point Form

# How to switch between these descriptions

Cartesian equation to Parametric equation: like you have practiced when solving linear systems

Cartesian equation to unit normal vector form: read off unit normal vector from the coefficients and find one solution of the equation

Unit normal vector form to Cartesian equation: the normal vector gives the coefficients, its dot product with the stationary vector gives the constant term.

Vector equation to unit normal vector: take the cross product of the direction vectors and normalize it (i.e., divide by its length).

Parametric equation to vector equation and back: very easy

Three point form to vector equation: subtract on of the vectors from the other two to get the direction vectors

Anywhere to three point form: Find three solutions that do not lie on a line

# Different ways to describe a line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix}$$

Vector Equation

Through $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ -2 \\ 4 \end{pmatrix}$

Two point form

$$\begin{aligned} x &= 2 + 7t \\ y &= 1 - 3t \\ z &= 3 + t \end{aligned}$$

Parametric Equation

$$\frac{x-2}{7} = -\frac{y-1}{3} = z - 3$$

Cartesian Equation

Switching is easy. To switch from a parametric equation for a line to a Cartesian equation for the same line, solve for $t$.

# Intersections

Whenever we ask you to find the intersection of two things, we are asking you to solve a linear system.

If you are trying to intersect two planes and the system has no solution, they are parallel (but not identical). Similarly, a line could be parallel to a plane or intersect it, two lines can either intersect or be parallel or skew, etc.

Careful: the definition of angle between two planes or two lines in the official slides is ambiguous.

For a linear system in three variables, our intuition for how many solutions we expect coincides with our intuition for intersecting planes in $\mathbb{R}^3$.